AP Statistics

Sample Student Responses and Scoring Commentary

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Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate probabilities associated with treatment and control group memberships for two different methods of random assignment and (2) justify which method of random assignment is more appropriate in a given situation.

Solution

Part (a):

(i) Let T (tail) represent being assigned to the treatment group and H (head) represent being assigned to the control group for each coin flip. The process stops when either the treatment group or the control group has two members. The outcomes and their probabilities are as follows.

Arrangement	А	В	С	D	E	F
Coin outcomes	TT	THT	THH	HH	HTH	HTT
Calculation	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
Probability	$\frac{1}{4}$	1 8	1/8	$\frac{1}{4}$	<u>1</u> 8	1/8

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

Part (b):

(i) Let T represent being assigned to the treatment group and C represent being assigned to the control group for each chip drawn. The process stops when either the treatment group or the control group has two members. The probabilities differ from the coin flip method because chips are drawn *without* replacement. The outcomes and their probabilities are as follows.

Arrangement	А	В	С	D	E	F
Chip outcomes	TT	TCT	TCC	CC	CTC	CTT
Calculation	$\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$
Probability	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u> 6	<u>1</u>

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$$

Question 6 (continued)

Part (c):

Use the chip method. The chip method gives equal probability to all possible arrangements, but the coin method does not, as shown in the tables from parts (a-i) and (b-i). Furthermore, the coin method is more likely to result in imbalanced treatment groups with regard to students and teachers, based on the probabilities in parts (a-ii) and (b-ii). If food preferences for teachers are different than for students, the imbalance is a problem. For example, if one treatment group consists entirely of students, it would be impossible to know if a difference in the response variable is due to the treatment (type of meal) or the role of the person at the school (teacher or student).

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following three components:

- 1. The correct probability for each arrangement in the table of (i).
- 2. Appropriate justification is shown in (i).
- 3. The correct probability is given in (ii).

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- For component 2, examples of appropriate justification include the following:
 - o Shows calculation of a correct probability for at least one of arrangements A and D AND shows calculation of a correct probability for at least one of arrangements B, C, E, and F. Calculations must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
 - Lists correct coin outcomes for at least one of arrangements A and D AND lists correct coin outcomes for at least one of arrangements B, C, E, and F (for example, A: TT and B: THT).
 Lists of outcomes must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
 - o Shows a tree diagram that ends with six branches corresponding to the six arrangements. Probabilities or coin outcomes must be listed on the branches, but arrangements do not need to be identified.
 - o Shows a tree diagram that ends with eight (or 16) branches where the branches leading to at least one of arrangements A and D are identified, and the branches leading to at least one of arrangements B, C, E, and F are identified. Probabilities or coin outcomes must be listed on the branches.
- If an incorrect probability is given in part (ii), component 3 is satisfied if the probability is between 0 and 1 and:
 - o is the result of a minor arithmetic error; OR
 - o is the result of adding the probabilities of arrangements A and D from the table in (i).

Question 6 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following two components:

- 1. The correct probability for each arrangement in the table of (i).
- 2. The correct probability is given in (ii).

Partially correct (P) if the response includes only one of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: If an incorrect probability is given in (ii), component 2 is satisfied if the probability is between 0 and 1 and

- is the result of a minor arithmetic error; OR
- is the result of adding the probabilities of arrangements A and D from the table in (i); OR
- work is shown that demonstrates the same incorrect approach is used in both parts (a-ii) and (b-ii). For example, component 2 is satisfied if the response forgets to include P(D) and says

 $P(A) = \frac{1}{4}$ in (a-ii) and $P(A) = \frac{1}{6}$ in part (b-ii). However, unlabeled answers of $\frac{1}{4}$ in (a-ii) and $\frac{1}{6}$ in (b-ii) would not satisfy component 2 because it is unclear if the same approach is being used.

Part (c) is scored as follows:

Essentially correct (E) if the response chooses the chip method AND includes the following three components:

- 1. Provides a statistical benefit to the chip method. For example, stating that all arrangements are equally likely.
- 2. Provides a statistical drawback to the coin method. For example, stating that the coin method is more likely to result in imbalanced treatment groups.
- 3. States that the responses (opinions, food preferences) of teachers and students might be different.

Partially correct if the response chooses the chip method and includes only one or two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- A correct comparative statement can be used to satisfy both components 1 and 2. For example, stating that the coin method is *more likely* to result in a disproportionate number of students in one treatment group, or that the chip method has a *greater chance* of providing roughly equivalent treatment groups.
- Benefits and drawbacks must be about the probabilities of the arrangements (from the tables in (i)), or the probabilities of imbalanced (balanced) groups (from the questions in (ii)), or both.
- If the response chooses the coin method, look back to see if the coin method should be the preferred method based on incorrect work from parts (a) and (b). If so, score part (c) using the corresponding three components.

Question 6 (continued)

- If the response says that it does not matter which method to use, look back to see if the tables are the same in parts (a-i) and (b-i) or if the probabilities are the same in parts (a-ii) and (b-ii). If so, part (c) is partially correct if the response justifies the decision by stating that the two tables or the two probabilities are the same. To be essentially correct, the response also needs to satisfy component 3.
- If the response does not choose a method, deduct one component from the number of correct components.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

One part essentially correct and one or two parts partially correct

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

OR

OR

No parts essentially correct and two parts partially correct

- (a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	В	С	D	Е	F
Probability	.25	.125	.125	.25	,125	,125

NE						
37XF	MI	M2	W1	W2	Total	
A	4.5	To each	11	11	.25	
B	,5 Treat	.5 Ctrl	.5 Total	1	.125	
C	5 tren	,5 Ctal	,5 otal	1	,125	
D	S	. 5 Ctri	1	√	.25	
E	.5	.5	.5 Cta	1	.125	
F	C+U -5	Trim Trim	Fram	1	,125	
l l	CTIL	-	The second secon			

1=Flipping stops

(ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

Groups where this happens: A, D

,50



The six arrangements are repeated below.

Arrange	ement A
Treatment	Control
Man 1	Woman 1
Man 2	Woman 2

Arrange	ement B
Treatment	Control
Man 1	Man 2
Woman 1	Woman 2

Arrangement C						
Treatment	Control					
Man 1	Man 2					
Woman 2	Woman 1					

Arrangement D					
Treatment	Control				
Woman 1	Man 1				
Woman 2	Man 2				

Arrange	ement E
Treatment	Control
Man 2	Man 1
Woman 2	Woman 1

Arrangement F				
Treatment	Control			
Man 2	Man 1			
Woman 1	Woman 2			

- (b) For the chip method, two chips are marked "treatment" and two chips are marked "control." Each person selects one chip at random without replacement.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	В	С	D	Е	F
Probability	.166	.166	.166	.166	-166	.166

	MI		MZ	2	W.	1	W2	Total	
A	1 2	T	-3	·T		1	1	16	
B	12	1	3	С	2	T	1	16	10
C	1 2	T	3	C	之	C	1	6	_
D	1	C	3	C		1	1	16	
E	1/2	C	2/3	T	13	C	1	16	_
F	2	C	23	T	23	T	~/	1/6	

(ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?





(c) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatments? Justify your choice.

The chip method should be used. In the SCF method, if order is not randomly assigned (which it is not in this case), participants in the same subset (students/teachers) have a greater chance of the receiving the same treatment. Due to this, there is a possibility that results will be non-representative of the entire school; for example, if teachers tend to like salad more than students, and the majority of the salads got to the students, the results will indicate that most people at the school dislike the salad, despite this not being the case. Since the SCF method carries a greater chance of this discrepancy occurring, it should not be used; the chip method, which gives all treatment combinations an equal chance of happening, should be used instead.

STATISTICS SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam. Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Consider an experiment in which two men and two women will be randomly assigned to either a treatment group or a control group in such a way that each group has two people. The people are identified as Man 1, Man 2, Woman 1, and Woman 2. The six possible arrangements are shown below.

Arrangement A			
Treatment	Control		
Man 1	Woman 1		
Man 2	Woman 2		

Arrang	ement B	
Treatment	Control	
Man 1 Woman 1	Man 2 Woman 2	

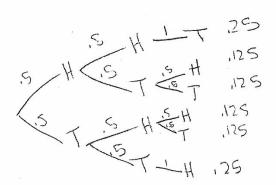
Arrange	ement C
Treatment	Control
Man 1 Woman 2	Man 2 Woman 1

Arrange	ement D	
Treatment	Control	
Woman 1	Man 1	
Woman 2	Man 2	

Arrange	ement E	
Treatment	Control	
Man 2 Woman 2	Man 1 Woman 1	

Arrangement F		
Treatment	Control	
Man 2 Woman 1	Man 1 Woman 2	

Two possible methods of assignment are being considered: the sequential coin flip method, as described in part (a), and the chip method, as described in part (b). For each method, the order of the assignment will be Man 1, Man 2, Woman 1, Woman 2.



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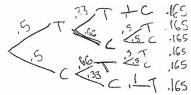
- (a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	В	С	· D	Е	F
Probability	,25	125	125	,25	125	125

(ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

The ecoloability that Man I and Man I are assigned to the same group is . 5.

The six arrangements are repeated below.



Arrange	ement A	
Treatment	Control	
Man 1	Woman 1	
Man 2 ·	Woman 2	

Arrange	ement B
Treatment	Control
Man 1	Man 2
Woman 1	Woman 2

Arrange	ement C
Treatment	Control
Man 1	Man 2
Woman 2	Woman 1

Arrange	ment D
Treatment	Control
Woman 1	Man 1
Woman 2	Man 2

Arrange	ement E
Treatment	Control
Man 2	Man 1
Woman 2	Woman 1

Arrange	ement F
Treatment	Control
Man 2 Man	
Woman 1	Woman 2

- (b) For the chip method, two chips are marked "treatment" and two chips are marked "control." Each person selects one chip at random without replacement.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	В	С	D	Е	F
Probability	: 1/G	1/6	1/6	1/6	1/6	1/6

(ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

The probability that man I and man 2 are assigned to the same group is 26 or .3333.

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(c) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatments? Justify your choice.

I would use the chip without because the probability of each arrangement is the same. If I use the coinstip method, a cossignment of their lunch will be independent on the people who went before them. And asher one treatment is silled up, the probability of any one getting that treatment of the that is O.

- (a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	В	С	D	Е	F
Probability	.25	ssis	.125	.25.	.125	.125

$$P(A) = P(T) \cdot P(T) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{4} = .35$$
 $P(B) = P(T) \cdot P(H) \cdot P(T) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{8} = .135$
 $P(C) = P(T) \cdot P(H) \cdot P(H) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3} = .135$
 $P(D) = P(H) \cdot P(H) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3} = .135$
 $P(E) = P(H) \cdot P(T) \cdot P(H) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3} = .135$
 $P(F) = P(H) \cdot P(T) \cdot P(T) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3} = .135$

 $\frac{\text{Key}}{P(\tau) = P(\tau a_i ls)}$ $P(+1) = P(H a_i ds)$

(ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

$$P(A) + P(D) = \frac{1}{4} + \frac{1}{4} = \frac{2}{3} = \frac{1}{3} = .25$$

the probability that man! and mana are assigned to the same group, using the sequential roin flip method is . 25

The six arrangements are repeated below.

Arrange	ement A
Treatment	Control
Man 1	Woman 1
Man 2	Woman 2

Arrange	ement B
Treatment	Control
Man 1	Man 2
Woman 1 Woman	

Arrange	ement C	
Treatment	Control	
Man 1	Man 2	
Woman 2	Woman 1	

Arrangement D		
Treatment	Control	
Woman 1	Man 1	
Woman 2	Man 2	

Arrange	ement E
Treatment	Control
Man 2	Man 1
Woman 2	Woman 1

Arrange	ement F
Treatment	Control
Man 2	Man 1
Woman 1	Woman 2

- (b) For the chip method, two chips are marked "treatment" and two chips are marked "control." Each person selects one chip at random without replacement.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	В	С	D	Е	F
Probability	.167	.167	.167	.167	.167	.167

$$P(A) = P(T, T, C, C) = \frac{a}{4} \cdot \frac{1}{3} \cdot \frac{a}{3} \cdot \frac{1}{4} = 1.167$$
 $P(B) = P(T, C, T, C) = \frac{a}{4} \cdot \frac{a}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1.167$
 $P(C) = P(T, C, C, T) = \frac{a}{4} \cdot \frac{a}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1.167$
 $P(E) = P(C, T, C, T) = \frac{a}{4} \cdot \frac{a}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1.167$
 $P(E) = P(C, T, T, C) = \frac{a}{4} \cdot \frac{a}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1.167$
 $P(F) = P(C, T, T, C) = \frac{a}{4} \cdot \frac{a}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1.167$

(ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

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(c) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatments? Justify your choice.

The Chip method should be used to determine assignments for 10 students and 6 teachers because the Chip method doesn't auto matically assign participants to another group once one group is

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AP® STATISTICS 2017 SCORING COMMENTARY

Question 6

Overview

The primary goals of this question were to assess a student's ability to (1) calculate probabilities associated with treatment and control group memberships for two different methods of random assignment and (2) justify which random assignment method is more appropriate in a given situation.

Sample: 6A Score: 4

In part (a-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. The response provides very good justification by showing correct calculations for each arrangement in an organized manner. Because the correct calculations are linked to the arrangements in the table with labels, component 2 is satisfied. Finally, the response gives the correct probability in part (a-ii), satisfying component 3. Because the response includes three components, part (a) was scored as essentially correct. In part (b-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. Although not required, the response provides very good justification for the probabilities by showing correct calculations for each arrangement. Finally, the response gives the correct probability in part (b-ii), satisfying component 2. Because the response includes both components, part (b) was scored as essentially correct. In part (c) the response correctly chooses the chip method. The response justifies the choice by stating that the sequential coin flip (SCF) method is more likely to produce imbalanced treatment groups because "participants in the same subset (students/teachers) have a greater chance of receiving the same treatment." Because the comparison between the two methods is correct, both component 1 and component 2 are satisfied. Furthermore, the response clearly explains that the imbalanced groups would cause a problem "if teachers tend to like salad more than students." The comparison of food preferences for students and teachers satisfies component 3. Because the response includes three components, part (c) was scored as essentially correct. Because three parts were scored as essentially correct, the response earned a score of 4.

Sample: 6B Score: 3

In part (a-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. The response provides justification by showing a reasonable six-branch tree diagram on the previous page. Labels for the arrangements are not required if the tree diagram has the appropriate six branches. Because the tree diagram includes the coin outcomes and the probabilities on the branches, component 2 is satisfied. Finally, the response gives the correct probability in part (a-ii), satisfying component 3. Because the response includes three components, part (a) was scored as essentially correct. In part (b-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. Although not required, the response provides good justification for the probabilities by showing a correct tree diagram at the top of the page. Finally, the response gives the correct probability in part (b-ii), satisfying component 2. Because the response includes both components, part (b) was scored as essentially correct. In part (c) the response correctly chooses the chip method. The response justifies the choice by stating that "the probability of each arrangement is the same." Because the statement provides a benefit of the chip method, component 1 is satisfied. The response also describes some additional facts about the coin method but never describes a drawback of the coin method that relates to the tables in part (i) or the probabilities in part (ii). Because no appropriate drawback of the coin method is provided, component 2 is not satisfied. Finally, because the response never explains that the preferences of teachers and students might differ, component 3 is not satisfied. Because the response includes only one of the three components, part (c) was scored as partially correct. Because two parts were scored as essentially correct. and one part was scored as partially correct, the response earned a score of 3.

AP® STATISTICS 2017 SCORING COMMENTARY

Question 6 (continued)

Sample: 6C Score: 2

In part (a-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. The response provides a very good justification by showing correct calculations for each arrangement, along with the corresponding coin outcomes. Because the correct calculations are linked to the arrangements in the table with labels, component 2 is satisfied. Finally, the response gives an incorrect probability in part (a-ii). However, the supporting work is correct, and the incorrect probability was identified as a minor arithmetic error. Therefore the response satisfies component 3. Because the response includes three components, part (a) was scored as essentially correct. In part (b-i) the response gives the correct probability for each arrangement in the table, satisfying component 1. Although not required, the response provides a very good justification for the probabilities by showing correct calculations for each arrangement. Finally, the response gives the correct probability in part (b-ii), satisfying component 2. Because the response includes both components, part (b) was scored as essentially correct. In part (c) the response correctly chooses the chip method. However, the response never describes a benefit of the chip method or a drawback of the coin method that relate to the tables in part (i) or the probabilities in part (ii). Consequently, neither component 1 nor component 2 is satisfied. Finally, because the response does not explain that the preferences of teachers and students might differ, component 3 is not satisfied. Because the response includes none of the three components, part (c) was scored as incorrect. Because two parts were scored as essentially correct, and one part was scored as incorrect, the response earned a score of 2.